

PERSONAL BEST TEST TRAINING

# MATH STUDY GUIDE

*ACT and PSAT/SAT and Other Standardized Tests*

 Doorway to College  
FOUNDATION  
Supporting the transition to higher education™

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## General Tips for a College-Admissions Test

### tip **There is only one correct answer to each item.**

**1** Professional test editors try to make sure that every item has one—and only one—correct answer. Even if two answer choices (called “foils” in the testing business) seem okay, there is probably a technical reason why one of those answers is wrong.

Do not waste time thinking about how a second answer could be defended. Professional editors do not intentionally create two correct answers, and the odds of a student winning an argument with ACT, Inc. or the College Board are quite remote. Besides, most people who can argue convincingly in favor of a wrong answer usually know which answer was considered correct in the first place. That is the answer you need to choose to get your best score. Don’t waste energy making mental protests while you’re taking the test.

### tip **Trick questions don’t show up on professional tests (not on purpose, anyway).**

**2** Professional editors like questions that are tricky, but not “trick questions.” Trick questions that get by the editors are nearly always thrown out after a field test. So don’t think the test is trying to trick you—not on purpose anyway.

The main point of math questions is to separate “good” students from “poor” students. Trick questions don’t help achieve this goal. Therefore, professional editors do not intentionally include trick questions on the test.

### tip **There is no identifiable pattern of correct answers.**

**3** Professional editors are very careful about making sure that the correct answers to their questions do not form a predictable pattern on the answer sheet. Even if there were a predictable pattern, only students achieving a perfect score would be able to discern it accurately. Even if you answered as few as one-fourth of the questions wrong, a perfect pattern would look totally random.

Some students look for patterns whenever they run into a guessing situation. “Let’s see now,” they reason, “I haven’t picked a D for about eight or nine questions, so I’ll guess D on this one.” Sound familiar? Don’t waste your time trying to track patterns because there simply aren’t any.

### tip **All choices are equally likely to occur.**

**4** On the tests that your teachers write, choice B or C will probably be correct more often than choices A or D. On professional tests, this idea does not hold true. The editors pay careful attention to the frequency of answers, and they change the order of choices whenever necessary. When you’re guessing, the main thing is to move quickly. A fast guess is just as good as a slow guess.

### tip **For you as an individual, the difficulty of the items is generally random.**

**5** You should always answer the easy questions first and skip over the harder ones. That way, if you need to guess on some items as time is running out, you’ll be guessing on the items that are the hardest ones for you. Otherwise, you might end up losing an easy point simply because you didn’t attempt all the easy items.

- 3) What is the sum of the measures of the angles of a triangle?
- 4) What is the formula for the area of a circle?
- 5) What is the formula for the circumference of a circle?
- 6) How many degrees are in the arc of a circle?
- 7) What is the formula for the area of a rectangle?
- 8) What is the formula for the volume of a box?
- 9) What is the formula for the volume of a cylinder?
- 10) Describe two special right triangles.

Answers are at the bottom of this page.



### Leave NO blanks!

It bears repeating: Do not, under any circumstances, leave an item blank. If you've got a lot left to do when you get the five-minute warning at the end of the test, quickly go back and randomly fill in answers for any blank items. If you find you have time at the end to work more problems, simply erase any incorrect guesses and replace them with the right answers.



### Check your work on every problem.

It's far more important to avoid careless mistakes than it is to finish all the items. Most students check their work only if they don't find their answer among the choices. This is a bad habit. As we've said before, the errors made on these items are often predictable. It's highly probable that you will find your wrong answer represented among the choices. Don't assume that your answer is correct just because you see it listed. Always, always check your work.



### Never create long or complex equations.

There is a straightforward method for solving every problem. If you find yourself in the middle of a long, drawn-out series of computations, you probably set up the problem incorrectly. Go back and rethink your approach to the problem.



### Avoid careless calculator mistakes.

Calculators can be the source of a lot of careless mistakes, especially if you aren't used to using one. Pay attention to the order of operations and know whether the calculator you are using has the order of operations built in. Probably the most common mistake is to plug in numbers in the same order they are given in the problem. This is almost guaranteed to give you the wrong answer.

Here is a brief review of the order of operations:

- 1) Work everything in parentheses *first*.
- 2) Simplify any square roots and exponents.
- 3) Perform any required multiplication and division (whichever comes first, from left to right).
- 4) Perform any required addition and subtraction (whichever comes first, from left to right).

## Tips for Open-Ended Items

"Open-ended" means you need to come up with the answer. These are not multiple-choice. There are a few of these on the SAT and PSAT. There are none on the ACT.

The most important task after figuring out the correct answer is filling in the answer grid correctly so the computer can score it. Don't laugh—for some students the grid is more challenging than the actual math. These open-ended items are sometimes called "gridded items."



### Do not grid-in mixed numbers.

The grid will accept fractions but *not* mixed numbers. An answer of  $4\frac{1}{4}$  should be gridded as  $17/4$  or 4.25. If it is gridded as  $4\ 1/4$ , the computer will read it as "forty-one fourths."



### There's no place for negatives or variables.

The grid is not designed to enter a negative answer or an answer with a variable in it. This means all of the grid-in items will be positive values—no negatives or variable expressions. If your answer is a negative number or contains a variable, go back and rethink how you worked the problem. You've definitely done something wrong.



### Don't bother reducing fractions.

A time saver! You don't have to reduce fractions to lowest terms; you just have to fit them into the grid (which can contain only four characters, including slashes).

So, if your answer is  $\frac{2}{8}$ , you do not have to reduce it to  $\frac{1}{4}$ . Either answer will be accepted.



### Be extra careful with repeating decimals.

If your answer turns out to be a repeating decimal or a decimal with more than three digits, include as much of the value in the grid as possible. You must give the most accurate answer that the grid will allow. In other words, use up all the spaces. For example, if your answer is .333333. . . , grid your answer as .333, not as .33. If your answer is .514285, grid your answer as .514, not .51.

# Chapter Three

## Content and Concepts Review

### Arithmetic and Numeration Concepts

#### Definitions

**Whole Numbers** 0, 1, 2, 3, ...

**Integers** ... -3, -2, -1, 0, 1, 2, 3, ...

**Digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

#### Divisibility

A number is divisible by:

- 2 if it ends in 0, 2, 4, 6, or 8.
- 3 if the sum of its digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if it ends in 0 or 5.
- 6 if it is divisible by both 2 and 3.
- 7 (sorry, no easy rule)
- 8 if the number formed by the last three digits is divisible by 8.
- 9 if the sum of its digits is divisible by 9.

#### The Order of Operations

You can use **PEMDAS** to help you remember the order of operations. (Or, you can simply remember this sentence: “**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.”)

1. **Parentheses.** Work everything inside parentheses first. Within the parentheses, follow the other rules.
2. **Exponents.** Simplify all powers and roots next.
3. **Multiplication and Division.** Perform any multiplication and division, from left to right.
4. **Addition and Subtraction.** Perform any addition and subtraction, from left to right.

#### Even and Odd Numbers

##### *Adding*

even + even = even

even + odd = odd

odd + odd = even

##### *Multiplying*

even × even = even

even × odd = even

odd × odd = odd

#### Absolute Value

The absolute value of a number is the number's distance from zero on a number line. Because absolute value is a distance, it is always a positive number. The absolute value of 0 is 0. The symbol for the absolute value of  $x$  is  $|x|$ .

$$|5| = 5 \quad |-5| = 5 \quad |0| = 0$$

#### Fractions

##### *Addition*

Fractions must have common denominators before you can add them. Find a common denominator. For each fraction, find the equivalent fraction that has the common denominator. Then add the numerators.

$$\text{Example: } \frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

##### *Subtraction*

Fractions must have common denominators before you can subtract them. Find a common denominator. For each fraction, find the equivalent fraction that has the common denominator. Then subtract the numerators.

$$\text{Example: } \frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

##### *Multiplication*

Fractions do NOT need common denominators before you can multiply them. Multiply the numerators, then multiply the denominators.

$$\text{Example: } \frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

##### *Division*

Fractions do NOT need to have common denominators before you can divide them. To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction. To find the reciprocal of a fraction, switch its numerator and denominator.

$$\text{Example: } \frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$



## Practice Set 313

1

$$(3y \cdot 4y^2 \cdot y^3)^2 = ?$$

- A)  $12y^7$
- B)  $12y^8$
- C)  $144y^9$
- D)  $144y^{12}$

2

The first term in a geometric sequence is 4. The next three terms, in order, are  $-2$ ,  $1$ , and  $-\frac{1}{2}$ . What is the eighth term in the sequence?

- A)  $\frac{1}{8}$
- B)  $\frac{1}{4}$
- C)  $-\frac{1}{8}$
- D)  $-\frac{1}{32}$

3

In  $\triangle ABC$ , the measure of  $\angle C$  is  $90^\circ$ . The length of  $\overline{AC}$  is 4 units, and the length of  $\overline{AB}$  is 8 units. What is the length of  $\overline{CB}$ ?

- A) 7
- B)  $4\sqrt{3}$
- C)  $4 + \sqrt{3}$
- D)  $4\sqrt{5}$

4

If four players on a basketball team are 6'1", 6'2", 6'3", and 6'5", how tall would a fifth player have to be so that the team average (arithmetic mean) would be 6'4"?

- A) 6'9"
- B) 6'8"
- C) 6'6"
- D) 6'5"

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- 6 **D** The cot  $A$  is the ratio of the side adjacent to  $A$ , which is 5, to the side opposite  $A$ , which is 7.  
The cot  $A = \frac{5}{7}$ .
- 7 **B** 4 hours and 33 minutes is  $4(60) + 33 = 273$  minutes. Since a sophomore spends 30% more time studying than a freshman, we have:  
 $1.3F = S$   
 $1.3F = 273$   
 $F = 210$   
 A freshman spends 210 minutes studying, which translates to 3 hours and 30 minutes.
- 8 **A** The total broadcast time is  $2(60) + 15 = 135$  minutes. Of that  $\frac{15}{27}$  will not be spent showing commercials, meaning that  $\frac{12}{27}$  will be spent showing commercials.  
 $\frac{12}{27}(135) = 60$ , so there are 60 minutes worth of commercials.  
 If each commercial only lasts 30 seconds (half a minute), there could potentially be  $60 \div 0.5 = 120$  commercials.  
 If each commercial lasts 45 seconds ( $\frac{3}{4}$  of a minute), there would only be  $60 \div 0.75 = 80$  commercials.  
 Thus, the difference between the maximum number of commercials and the minimum number of commercials is  $120 - 80 = 40$ .
- 9 **C** The expression is undefined whenever  $x^2 - 2x - 8 = 0$ . Factoring, we have:  
 $(x - 4)(x + 2) = 0$   
 Thus, the expression is undefined when  $x = 4$  and when  $x = -2$ .
- 10 **A** There are 58 7th graders who participate in 0–2 (i.e., fewer than 3 activities) out of 86 total 7th graders surveyed, giving a probability of  $\frac{58}{86} = \frac{29}{43}$ .
- 11 **D** Since 24% of juniors play a sport, 76% of juniors don't play a sport. Similarly, 80% of seniors don't play a sport. 76% of 150 is 114 juniors who don't play a sport, and 80% of 135 is 108 seniors who don't play a sport. So, a total of  $114 + 108 = 222$  upperclassmen don't play a sport, out of 285 total upperclassmen.  $222 \div 285 \approx 0.7789$ , approximately 78%.
- 12 **B** Here, we need to use the trigonometric identity  $\sin a^\circ = \cos(90 - a)^\circ$ , giving us  $a = 90 - b$ .  
 Since you're asked to find " $a$ ":  
 $b = 90 - a$   
 Thus,  $a + 5 = 2(90 - a) - 13$   
 $a + 5 = 180 - 2a - 13$   
 Now, subtract 5 and add  $2a$  on each side of the equation.  
 $3a = 175 - 13$   
 $3a = 162$   
 $a = 54$